GCE AS/A level

0983/01

MATHEMATICS – S1
Statistics

A.M. MONDAY, 10 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. The events A and B are such that

$$P(A) = 0{\cdot}25,\ P(A \cup B) = 0{\cdot}4.$$

Evaluate $P(B)$ when

(a) A, B are mutually exclusive, [2]

(b) A, B are independent. [3]

2. Simon has 3 types of DVDs; 5 war films, 3 cowboy films and 2 horror films. He selects 3 of the DVDs at random to watch one evening. Calculate the probability that he selects

(a) 1 film of each type, [3]

(b) 3 war films, [2]

(c) 3 films all of the same type. [3]

3. The random variable X has a binomial distribution with parameters $n = 25$, $p = 0{\cdot}8$. The random variable Y is defined by $Y = aX + b$, where $a, b > 0$.
Given that the mean and standard deviation of Y are 65 and 6 respectively, find the values of a and b. [6]

4. Bethan has two fair dice, each in the shape of a regular tetrahedron. The four faces of each dice are numbered 1, 2, 3, 4 respectively.

(a) She throws one of the dice 20 times and her score on each throw is defined as the number appearing on the face in contact with the table. Let X denote the number of throws resulting in a score of 4.

(i) Write down the distribution of X.

(ii) Determine $P(3 \leqslant X \leqslant 9)$.

(iii) **Without the use of tables**, calculate $P(X = 6)$. [6]

(b) She now throws the two dice simultaneously 160 times and her score on each throw is defined as the sum of the numbers on the two faces in contact with the table. Use a Poisson approximation to determine the probability that the number of throws resulting in a score of 8 is

(i) equal to 12,

(ii) between 6 and 14 (both inclusive). [6]

5. Box A contains four balls numbered 1, 2, 3, 4 respectively, Box B contains three balls numbered 1, 2, 3 respectively and Box C contains two balls numbered 1, 2 respectively. Gwen selects one of these boxes at random and then selects a ball at random from that box.

(a) Determine the probability that a ball numbered 1 is selected. [3]

(b) Given that a ball numbered 1 is selected, determine the probability that Box A was selected. [3]

6. When Mike fires his gun at a target, he hits it with probability 0·7. Successive shots are independent. When he starts to fire his gun at the target, calculate the probability that he hits the target

(a) for the first time on his fourth shot, [3]

(b) for the second time on his third shot. [3]

7. The probability distribution of the discrete random variable X is given by

$$P(X = x) = \frac{k}{x} \qquad \text{for } x = 1, 2, 4, 8,$$

$$P(X = x) = 0 \qquad \text{otherwise.}$$

(a) Show that $k = \frac{8}{15}$. [2]

(b) Determine the mean and variance of X. [5]

(c) Given that X_1, X_2 are independent observations on X,

(i) find the value of $P(X_1 = X_2)$,

(ii) use your answer to (i) to deduce the value of $P(X_1 > X_2)$. [5]

8. The number of telephone calls received in a certain office during the day in an interval of duration t hours can be modelled by a Poisson distribution with mean $5t$.
Without the use of tables, calculate

(a) the probability of receiving 7 telephone calls between 9 a.m. and 10 a.m., [3]

(b) the probability of receiving 7 telephone calls between 9 a.m. and 10 a.m., given that 10 telephone calls are received between 9 a.m. and 11 a.m. [5]

TURN OVER

9. The continuous random variable X has probability density function f given by

$$f(x) = k\left(1 - \frac{x^2}{4}\right) \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$f(x) = 0 \quad \text{otherwise.}$$

(a) Show that $k = \frac{3}{4}$. [3]

(b) Calculate $E(X)$. [4]

(c) (i) Find an expression for $F(x)$, valid for $0 \leqslant x \leqslant 2$, where F denotes the cumulative distribution function of X.

(ii) Hence evaluate $P(0{\cdot}5 \leqslant X \leqslant 1{\cdot}5)$. [5]